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Analysis of $\Omega_c^*(css)$ and $\Omega_b^*(bss)$ with QCD sum rules

Zhi-Gang Wang[a]

Department of Physics, North China Electric Power University, Baoding 071003, P.R. China

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Abstract. In this article, we calculate the masses and residues of the heavy baryons $\Omega_c^*(css)$ and $\Omega_b^*(bss)$ with spin–parity $\frac{3}{2}^+$ with the QCD sum rules. The numerical values are compatible with the experimental data and other theoretical estimations.

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1 Introduction

Several new excited charmed baryon states have been observed by the BaBar, Belle and CLEO Collaborations, such as $\Lambda_c(2765)^+$, $\Lambda_c^+(2880)$, $\Lambda_c^+(2940)$, $\Sigma_c^+(2800)$, $\Xi_c^+(2980)$, $\Xi_c^+(3077)$, $\Xi_c^0(2980)$, $\Xi_c^0(3077)$ [1–7]. The charmed baryons provide a rich source of states, including possible candidates for the orbital excitations. They serve as an excellent ground for testing the predictions of the constituent quark models and heavy quark symmetry [8, 9]. The charmed and bottomed baryons, which contain a heavy quark and two light quarks, provide an ideal tool for studying the dynamics of the light quarks in the presence of a heavy quark. The u, d and s quarks form an SU(3) flavor triplet, $3\times 3=\bar{3}+6$, two light quarks can form diquarks with a symmetric sextet and an antisymmetric antitriplet. For the S-wave baryons, the sextet contains both spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ states, while the antitriplet contains only spin-$\frac{1}{2}$ states. By now, the $\frac{1}{2}^+$ antitriplet states (Λ_c^+, Ξ_c^+,Ξ_c^0), and the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ sextet states ($\Omega_c,\Sigma_c,\Xi_c^{\prime}$) and ($\Omega_c^*,\Sigma_c^*,\Xi_c^{\prime *}$) have been established.

The baryon Ω_c^*, a css candidate for the $\frac{3}{2}$ partner of the strange baryon $\Omega(sss)$, was observed by the BaBar Collaboration in the radiative decay $\Omega_c^*\to\Omega_c\gamma$ [10]. The $\frac{1}{2}^+$ baryon $\Omega_c(css)$ was reconstructed in decays to the final states $\Omega^-\pi^+$, $\Omega^-\pi^+\pi^0$, $\Omega^-\pi^+\pi^-\pi^+$ and $\Xi^-K^-\pi^+\pi^+$. It lies about $70.8\pm1.0\pm1.1$ MeV above the Ω_c, and it is the last singly-charmed baryon with zero orbital momentum observed experimentally [11].

In this article, we calculate the mass and residue of the Ω_c^* (and Ω_b^* as byproduct; the Ω_b^* has not been observed experimentally yet) with the QCD sum rule [12–15] approach. In the QCD sum rule approach, an operator product expansion is used to expand the time-ordered currents into a series of quark and gluon condensates that parameterize the long distance properties of the QCD vacuum. Based on current–hadron duality, we can obtain copious information about the hadronic parameters on the phenomenological side.

The article is arranged as follows: we derive the QCD sum rules for the masses and residues of the Ω_c^* and Ω_b^* in Sect. 2; in Sect. 3, numerical results and discussions; Section 4 is reserved for our conclusions.

2 QCD sum rules for the Ω_c^* and Ω_b^*

In the following, we write down the two-point correlation functions $\Pi_{\mu\nu}^a(p^2)$ in the QCD sum rule approach,

$$\Pi_{\mu\nu}^a(p^2)=\mathrm{i}\int \mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}p\cdot x}\langle 0|T\left\{J_\mu^a(x)\bar{J}_\nu^a(0)\right\}|0\rangle\,, \quad (1)$$

$$J_\mu^a(x)=\epsilon_{ijk}s_i^{\mathrm{T}}(x)C\gamma_\mu s_j(x)Q_k^a(x)\,, \quad (2)$$

$$\lambda_a N_\mu(p,s)=\left\langle 0|J_\mu^a(0)\middle|\Omega_a^*(p,s)\right\rangle\,, \quad (3)$$

where the upper index a represents the c and b quarks respectively; the $N_\mu(p,s)$ and λ_a stand for the Rarita–Schwinger spin vector and residue of the baryon Ω_a^*, respectively. i, j and k are color indexes, C is the charge conjunction matrix, and μ and ν are Lorentz indexes.

The correlation functions $\Pi_{\mu\nu}^a(p)$ can be decomposed as follows:

$$\Pi_{\mu\nu}^a(p)=-g_{\mu\nu}\left\{\not{p}\Pi_1^a(p^2)+\Pi_2^a(p^2)\right\}+\cdots\,, \quad (4)$$

due to Lorentz covariance. The first structure $g_{\mu\nu}\not{p}$ has an odd number of γ-matrices and conserves chirality; the second structure $g_{\mu\nu}$ has an even number of γ-matrices and violates chirality. In the original QCD sum rule analysis of the nucleon masses and magnetic moments [16–19], the interval of dimensions (of the condensates) for the odd structure is larger than the interval of dimensions for the even

[a] e-mail: wangzgyiti@yahoo.com.cn

structure, and one may expect a better accuracy of the results obtained from the sum rules with the odd structure.

In this article, we choose the two tensor structures to study the masses and residues of the heavy baryons Ω_c^* and Ω_b^*, as the masses of the heavy quarks break the chiral symmetry explicitly.

According to the basic assumption of current–hadron duality in the QCD sum rule approach [12, 13], we insert a complete series of intermediate states satisfying the unitarity principle with the same quantum numbers as the current operator $J^a_\mu(x)$ into the correlation functions in (1) to obtain the hadronic representation. After isolating the pole terms of the lowest states Ω_a^*, we obtain the following result:

$$\Pi^a_{\mu\nu}(p^2) = -g_{\mu\nu}\lambda_a^2 \frac{M_{\Omega_a^*}+\not p}{M^2_{\Omega_a^*}-p^2}+\cdots\,, \quad (5)$$

where we have used this relation to sum over the Rarita–Schwinger spin vector:

$$\sum_s N_\mu(p,s)\bar N_\nu(p,s) = -(\not p + M_{\Omega_a^*}) \times\left\{g_{\mu\nu}-\frac{\gamma_\mu\gamma_\nu}{3}-\frac{2p_\mu p_\nu}{3M^2_{\Omega_a^*}}+\frac{p_\mu\gamma_\nu-p_\nu\gamma_\mu}{3M_{\Omega_a^*}}\right\}. \quad (6)$$

In the following, we briefly outline the operator product expansion for the correlation functions $\Pi^a_{\mu\nu}(p)$ in perturbative QCD theory. The calculations are performed at large space-like momentum in the region $p^2 \ll 0$, which corresponds to the small distance $x\approx 0$ required by the validity of the operator product expansion. We write down the "full" propagators $S_{ij}(x)$ and $S_Q^{ij}(x)$ of a massive quark in the presence of the vacuum condensates firstly [12, 13][1],

$$\begin{aligned} S_{ij}(x) =& \frac{\mathrm{i}\delta_{ij}\not x}{2\pi^2x^4}-\frac{\delta_{ij}m_s}{4\pi^2x^2}-\frac{\delta_{ij}}{12}\langle\bar ss\rangle+\frac{\mathrm{i}\delta_{ij}}{48}m_s\langle\bar ss\rangle\not x \\ &-\frac{\delta_{ij}x^2}{192}\langle\bar sg_s\sigma Gs\rangle+\frac{\mathrm{i}\delta_{ij}x^2}{1152}m_s\langle\bar sg_s\sigma Gs\rangle\not x \\ &-\frac{\mathrm{i}}{32\pi^2x^2}G^{ij}_{\mu\nu}(\not x\sigma^{\mu\nu}+\sigma^{\mu\nu}\not x)+\cdots\,, \\ S_Q^{ij}(x) =& \frac{\mathrm{i}}{(2\pi)^4}\int \mathrm{d}^4k\,\mathrm{e}^{-\mathrm{i}kx} \\ &\times\left\{\frac{\delta_{ij}}{\not k-m_Q}-\frac{g_sG^{\alpha\beta}_{ij}}{4}\frac{\sigma_{\alpha\beta}(\not k+m_Q)+(\not k+m_Q)\sigma_{\alpha\beta}}{\left(k^2-m_Q^2\right)^2}\right. \\ &\left.+\frac{\pi^2}{3}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\delta_{ij}m_Q\frac{k^2+m_Q\not k}{\left(k^2-m_Q^2\right)^4}+\cdots\right\}, \end{aligned} \quad (7)$$

where $\langle\bar sg_s\sigma Gs\rangle=\langle\bar sg_s\sigma_{\alpha\beta}G^{\alpha\beta}s\rangle$ and $\left\langle\frac{\alpha_sGG}{\pi}\right\rangle=\left\langle\frac{\alpha_sG_{\alpha\beta}G^{\alpha\beta}}{\pi}\right\rangle$, then contract the quark fields in the correlation functions $\Pi^a_{\mu\nu}(p)$ with the Wick theorem, and obtain the result

$$\Pi^a_{\mu\nu}(p) = 2\mathrm{i}\epsilon_{ijk}\epsilon_{i'j'k'}\int \mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}px} \times \mathrm{Tr}\left\{\gamma_\mu S_{ii'}(x)\gamma_\nu CS^{\mathrm{T}}_{jj'}(x)C\right\}S_Q^{kk'}(x)\,. \quad (8)$$

Substituting the full s, c and b quark propagators into the above correlation functions and completing the integral in coordinate space, then integrating over the variable k, we can obtain the correlation functions $\Pi^a_i(p^2)$ at the level of quark–gluon degree of freedom:

$$\begin{aligned} \Pi_1^a(p^2) =& -\frac{1}{64\pi^4}\int_0^1 \mathrm{d}x x(1-x)^2(x+2) \\ &\times\left(\widetilde m_a^2-p^2\right)^2\log\left(\widetilde m_a^2-p^2\right) \\ &-\frac{m_s\langle\bar ss\rangle}{4\pi^2}\int_0^1 \mathrm{d}x x(x-2)\log\left(\widetilde m_a^2-p^2\right) \\ &+\frac{1}{192\pi^2}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\int_0^1 \mathrm{d}x x(x-2)\log\left(\widetilde m_a^2-p^2\right) \\ &+\frac{m_s\langle\bar sg_s\sigma Gs\rangle}{24\pi^2}\int_0^1 \mathrm{d}x\frac{x}{\widetilde m_a^2-p^2}-\frac{m_a^2}{576\pi^2} \\ &\times\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\int_0^1 \mathrm{d}x\frac{(1-x)^2(x+2)}{x^2(\widetilde m_a^2-p^2)} \\ &+\frac{\langle\bar ss\rangle^2}{3}\frac{1}{m_a^2-p^2}+\frac{m_s\langle\bar sg_s\sigma Gs\rangle}{12\pi^2}\frac{1}{m_a^2-p^2}+\cdots\,, \end{aligned} \quad (9)$$

$$\begin{aligned} \Pi_2^a(p^2) =& -\frac{m_a}{64\pi^4}\int_0^1 \mathrm{d}x(1-x)^2(x+2) \\ &\times\left(\widetilde m_a^2-p^2\right)^2\log\left(\widetilde m_a^2-p^2\right)-\frac{m_am_s\langle\bar ss\rangle}{4\pi^2} \\ &\times\int_0^1 \mathrm{d}x(x-2)\log\left(\widetilde m_a^2-p^2\right)-\frac{m_a}{576\pi^2}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle \\ &\times\int_0^1 \mathrm{d}x\left(\frac{4}{x^2}-\frac{9}{x}-3x^2+2x+9\right) \\ &\times\log\left(\widetilde m_a^2-p^2\right)+\frac{m_a}{576\pi^2}\left\langle\frac{\alpha_sGG}{\pi}\right\rangle \\ &\times\int_0^1 \mathrm{d}x\frac{x^4-x^3-3x^2+5x-2}{x(1-x)}\frac{\widetilde m_a^2}{\widetilde m_a^2-p^2} \\ &+\frac{m_a\,m_s\langle\bar sg_s\sigma Gs\rangle}{24\pi^2} \\ &\times\int_0^1 \mathrm{d}x\frac{1}{\widetilde m_a^2-p^2}+\frac{m_a\langle\bar ss\rangle^2}{3}\frac{1}{m_a^2-p^2} \\ &+\frac{m_am_s\langle\bar sg_s\sigma Gs\rangle}{12\pi^2}\frac{1}{m_a^2-p^2}+\cdots\,, \end{aligned} \quad (10)$$

where $\widetilde m_a^2=\frac{m_a^2}{x}$.

We carry out the operator product expansion to the vacuum condensates adding up to dimension-6. In the calculation, we make the assumption of vacuum saturation for high dimension vacuum condensates; they are always factorized to lower condensates with vacuum saturation in the QCD sum rules, and factorization works well in the large N_c limit. In this article, we take into account the contri-

[1] One may consult the last article of [12, 13] for technical details in deriving the full propagator.

butions from the quark condensate $\langle\bar{s}s\rangle$, the mixed condensate $\langle\bar{s}g_s\sigma Gs\rangle$, the gluon condensate $\langle\frac{\alpha_s GG}{\pi}\rangle$, and we neglect the contributions from other high dimension condensates, which are suppressed by large denominators and would not play significant roles.

Once analytical results are obtained, we can take current–hadron duality below the threshold s_a^0 and perform a Borel transformation with respect to the variable $P^2=-p^2$; finally we obtain the following sum rules:

$$\begin{aligned}
&\lambda_a^2 \exp\left\{-\frac{M_{\Omega_a^*}^2}{M^2}\right\} \\
&= \frac{1}{64\pi^4}\int_{\mathrm{th}}^{s_a^0} \mathrm{d}s \int_{\Delta^a}^1 \mathrm{d}x x(1-x)^2(x+2) \\
&\quad\times \left(\widetilde{m}_a^2-s\right)^2 \exp\left\{-\frac{s}{M^2}\right\} + \frac{m_s\langle\bar{s}s\rangle}{4\pi^2} \\
&\quad\times \int_{\mathrm{th}}^{s_a^0} \mathrm{d}s \int_{\Delta^a}^1 \mathrm{d}x x(x-2)\exp\left\{-\frac{s}{M^2}\right\} \\
&\quad- \frac{1}{192\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \int_{\mathrm{th}}^{s_a^0} \mathrm{d}s \int_{\Delta^a}^1 \mathrm{d}x x(x-2)\exp\left\{-\frac{s}{M^2}\right\} \\
&\quad+ \frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{24\pi^2}\int_0^1 \mathrm{d}x x \exp\left\{-\frac{\widetilde{m}_a^2}{M^2}\right\} \\
&\quad- \frac{m_a^2}{576\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \int_0^1 \mathrm{d}x \frac{(1-x)^2(2+x)}{x^2}\exp\left\{-\frac{\widetilde{m}_a^2}{M^2}\right\} \\
&\quad+ \frac{\langle\bar{s}s\rangle^2}{3}\exp\left\{-\frac{m_a^2}{M^2}\right\} + \frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{12\pi^2}\exp\left\{-\frac{m_a^2}{M^2}\right\},
\end{aligned} \tag{11}$$

$$\begin{aligned}
&M_{\Omega_a^*}\lambda_a^2 \exp\left\{-\frac{M_{\Omega_a^*}^2}{M^2}\right\} \\
&= \frac{m_a}{64\pi^4}\int_{\mathrm{th}}^{s_a^0} \mathrm{d}s \int_{\Delta^a}^1 \mathrm{d}x(1-x)^2(x+2) \\
&\quad\times \left(\widetilde{m}_a^2-s\right)^2 \exp\left\{-\frac{s}{M^2}\right\} + \frac{m_a m_s\langle\bar{s}s\rangle}{4\pi^2} \\
&\quad\times \int_{\mathrm{th}}^{s_a^0} \mathrm{d}s \int_{\Delta^a}^1 \mathrm{d}x(x-2)\exp\left\{-\frac{s}{M^2}\right\} + \frac{m_a}{576\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\quad\times \int_{\mathrm{th}}^{s_a^0} \mathrm{d}s \int_{\Delta^a}^1 \mathrm{d}x\left(\frac{4}{x^2}-\frac{9}{x}-3x^2+2x+9\right)\exp\left\{-\frac{s}{M^2}\right\} \\
&\quad+ \frac{m_a}{576\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\quad\times \int_0^1 \mathrm{d}x \frac{x^4-x^3-3x^2+5x-2}{x(1-x)}\widetilde{m}_a^2 \exp\left\{-\frac{\widetilde{m}_a^2}{M^2}\right\} \\
&\quad+ \frac{m_a\, m_s\langle\bar{s}g_s\sigma Gs\rangle}{24\pi^2} \\
&\quad\times \int_0^1 \mathrm{d}x \exp\left\{-\frac{\widetilde{m}_a^2}{M^2}\right\} + \frac{m_a\langle\bar{s}s\rangle^2}{3}\exp\left\{-\frac{m_a^2}{M^2}\right\} \\
&\quad+ \frac{m_a m_s\langle\bar{s}g_s\sigma Gs\rangle}{12\pi^2}\exp\left\{-\frac{m_a^2}{M^2}\right\},
\end{aligned} \tag{12}$$

where $th=(m_a+2m_s)^2$ and $\Delta^a=\frac{m_a^2}{s}$.

Differentiating the above sum rules with respect to the variable $\frac{1}{M^2}$, then eliminating the quantity $\lambda_{\Omega_a^*}$, we obtain two QCD sum rules for the masses $M_{\Omega_a^*}$:

$$\begin{aligned}
M_{\Omega_a^*}^2 = &\left[\frac{1}{64\pi^4}\int_{\mathrm{th}}^{s_a^0} \mathrm{d}s \int_{\Delta^a}^1 \mathrm{d}x x(1-x)^2(x+2)\right. \\
&\times \left(\widetilde{m}_a^2-s\right)^2 s\exp\left\{-\frac{s}{M^2}\right\} \\
&+ \frac{m_s\langle\bar{s}s\rangle}{4\pi^2}\int_{\mathrm{th}}^{s_a^0} \mathrm{d}s \int_{\Delta^a}^1 \mathrm{d}x x(x-2)s\exp\left\{-\frac{s}{M^2}\right\} \\
&- \frac{1}{192\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int_{\mathrm{th}}^{s_a^0} \mathrm{d}s \int_{\Delta^a}^1 \mathrm{d}x x(x-2)s\exp\left\{-\frac{s}{M^2}\right\} \\
&+ \frac{m_s m_a^2\langle\bar{s}g_s\sigma Gs\rangle}{24\pi^2}\int_0^1 \mathrm{d}x \exp\left\{-\frac{\widetilde{m}_a^2}{M^2}\right\} \\
&- \frac{m_a^4}{576\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int_0^1 \mathrm{d}x \frac{(1-x)^2(2+x)}{x^3}\exp\left\{-\frac{\widetilde{m}_a^2}{M^2}\right\} \\
&+ \frac{m_a^2\langle\bar{s}s\rangle^2}{3}\exp\left\{-\frac{m_a^2}{M^2}\right\} \\
&\left.+ \frac{m_s m_a^2\langle\bar{s}g_s\sigma Gs\rangle}{12\pi^2}\exp\left\{-\frac{m_a^2}{M^2}\right\}\right] \\
&\Big/ \left[\frac{1}{64\pi^4}\int_{\mathrm{th}}^{s_a^0} \mathrm{d}s \int_{\Delta^a}^1 \mathrm{d}x x(1-x)^2(x+2)\right. \\
&\times \left(\widetilde{m}_a^2-s\right)^2 \exp\left\{-\frac{s}{M^2}\right\} \\
&+ \frac{m_s\langle\bar{s}s\rangle}{4\pi^2}\int_{\mathrm{th}}^{s_a^0} \mathrm{d}s \int_{\Delta^a}^1 \mathrm{d}x x(x-2)\exp\left\{-\frac{s}{M^2}\right\} \\
&- \frac{1}{192\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int_{\mathrm{th}}^{s_a^0} \mathrm{d}s \int_{\Delta^a}^1 \mathrm{d}x x(x-2)\exp\left\{-\frac{s}{M^2}\right\} \\
&+ \frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{24\pi^2}\int_0^1 \mathrm{d}x x \exp\left\{-\frac{\widetilde{m}_a^2}{M^2}\right\} \\
&- \frac{m_a^2}{576\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \\
&\times \int_0^1 \mathrm{d}x \frac{(1-x)^2(x+2)}{x^2}\exp\left\{-\frac{\widetilde{m}_a^2}{M^2}\right\} + \frac{\langle\bar{s}s\rangle^2}{3} \\
&\left.\times \exp\left\{-\frac{m_a^2}{M^2}\right\} + \frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{12\pi^2}\exp\left\{-\frac{m_a^2}{M^2}\right\}\right],
\end{aligned} \tag{13}$$

and

$$\begin{aligned}
M_{\Omega_a^*}^2 = &\left[\frac{m_a}{64\pi^4}\int_{\mathrm{th}}^{s_a^0} \mathrm{d}s \int_{\Delta^a}^1 \mathrm{d}x(1-x)^2(x+2)\right. \\
&\times \left(\widetilde{m}_a^2-s\right)^2 s\exp\left\{-\frac{s}{M^2}\right\}
\end{aligned}$$

$$
\begin{aligned}
&+\frac{m_a m_s\langle\bar{s}s\rangle}{4\pi^2}\int_{\rm th}^{s_a^0} {\rm d}s\int_{\Delta^a}^1 {\rm d}x(x-2)s\exp\left\{-\frac{s}{M^2}\right\}\\
&+\frac{m_a}{576\pi^2}\left\langle\frac{\alpha_{\rm s}GG}{\pi}\right\rangle\\
&\times\int_{\rm th}^{s_a^0}{\rm d}s\int_{\Delta^a}^1{\rm d}x\left(\frac{4}{x^2}-\frac{9}{x}-3x^2+2x+9\right)s\\
&\times\exp\left\{-\frac{s}{M^2}\right\}+\frac{m_a}{576\pi^2}\left\langle\frac{\alpha_{\rm s}GG}{\pi}\right\rangle\\
&\times\int_0^1{\rm d}x\frac{x^4-x^3-3x^2+5x-2}{x(1-x)}\widetilde{m}_a^4\exp\left\{-\frac{\widetilde{m}_a^2}{M^2}\right\}\\
&+\frac{m_a^3 m_s\langle\bar{s}g_s\sigma Gs\rangle}{24\pi^2}\\
&\times\int_0^1{\rm d}x\frac{1}{x}\exp\left\{-\frac{\widetilde{m}_a^2}{M^2}\right\}+\frac{m_a^3\langle\bar{s}s\rangle^2}{3}\exp\left\{-\frac{m_a^2}{M^2}\right\}\\
&+\frac{m_a^3 m_s\langle\bar{s}g_s\sigma Gs\rangle}{12\pi^2}\exp\left\{-\frac{m_a^2}{M^2}\right\}\Bigg]\\
&\Bigg/\Bigg[\frac{m_a}{64\pi^4}\int_{\rm th}^{s_a^0}{\rm d}s\int_{\Delta^a}^1{\rm d}x(1-x)^2(x+2)\\
&\times\left(\widetilde{m}_a^2-s\right)^2\exp\left\{-\frac{s}{M^2}\right\}\\
&+\frac{m_a m_s\langle\bar{s}s\rangle}{4\pi^2}\int_{\rm th}^{s_a^0}{\rm d}s\int_{\Delta^a}^1{\rm d}x(x-2)\exp\left\{-\frac{s}{M^2}\right\}\\
&+\frac{m_a}{576\pi^2}\left\langle\frac{\alpha_{\rm s}GG}{\pi}\right\rangle\\
&\times\int_{\rm th}^{s_a^0}{\rm d}s\int_{\Delta^a}^1{\rm d}x\left(\frac{4}{x^2}-\frac{9}{x}-3x^2+2x+9\right)\\
&\times\exp\left\{-\frac{s}{M^2}\right\}+\frac{m_a}{576\pi^2}\left\langle\frac{\alpha_{\rm s}GG}{\pi}\right\rangle\\
&\times\int_0^1{\rm d}x\frac{x^4-x^3-3x^2+5x-2}{x(1-x)}\widetilde{m}_a^2\exp\left\{-\frac{\widetilde{m}_a^2}{M^2}\right\}\\
&+\frac{m_a\, m_s\langle\bar{s}g_s\sigma Gs\rangle}{24\pi^2}\\
&\times\int_0^1{\rm d}x\exp\left\{-\frac{\widetilde{m}_a^2}{M^2}\right\}+\frac{m_a\langle\bar{s}s\rangle^2}{3}\exp\left\{-\frac{m_a^2}{M^2}\right\}\\
&+\frac{m_a m_s\langle\bar{s}g_s\sigma Gs\rangle}{12\pi^2}\exp\left\{-\frac{m_a^2}{M^2}\right\}\Bigg]. \qquad (14)
\end{aligned}
$$

3 Numerical results and discussions

The input parameters are taken to be the standard values $\langle\bar{q}q\rangle=-(0.24\pm0.01\,\text{GeV})^3$, $\langle\bar{s}s\rangle=(0.8\pm0.2)\langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle=m_0^2\langle\bar{s}s\rangle$, $m_0^2=(0.8\pm0.2)\,\text{GeV}^2$, $\langle\frac{\alpha_{\rm s}GG}{\pi}\rangle=(0.33\,\text{GeV})^4$, $m_s=(0.14\pm0.01)\,\text{GeV}$, $m_c=(1.4\pm0.1)\,\text{GeV}$ and $m_b=(4.8\pm0.1)\,\text{GeV}$ [12–15, 20]. The contribution from the gluon condensate $\langle\frac{\alpha_{\rm s}GG}{\pi}\rangle$ is less than 4%, and the uncertainty is neglected here.

For the octet baryons with $I(J^P)=\frac{1}{2}(\frac{1}{2}^+)$, the mass of the proton (the ground state) is $M_p=938\,\text{MeV}$, and the mass of the first radial excited state $N(1440)$ (the Roper resonance) is $M_{1440}=(1420\text{–}1470)\,\text{MeV}\approx1440\,\text{MeV}$ [21]. For the decuplet baryons with $I(J^P)=\frac{3}{2}(\frac{3}{2}^+)$, the mass of the $\Delta(1232)$ (the ground state) is $M_{1232}=(1231\text{–}1233)\,\text{MeV}\approx1232\,\text{MeV}$, and the mass of the first radial excited state $\Delta(1600)$ is $M_{1600}=(1550\text{–}1700)\,\text{MeV}\approx1600\,\text{MeV}$ [21]. The separation between the ground states and first radial excited states is about 0.5 GeV. So in the QCD sum rules for the baryons with light quarks, the threshold parameters s_0 are always chosen to be $\sqrt{s_0}=M_{\rm gr}+0.5\,\text{GeV}$ [16–19, 22–24], where gr stands for the ground states. The threshold parameters for the heavy baryons Ω_c^* and Ω_b^* can be chosen to be $s^0_{\Omega_c^*}=(2.8+0.5)^2\,\text{GeV}^2$ and $s^0_{\Omega_b^*}=(6.1+0.5)^2\,\text{GeV}^2$, respectively. The mass of the bottomed baryon Ω_b^* with spin–parity $\frac{3}{2}^+$ is about $M_{\Omega_b^*}=(6.04-6.09)\,\text{GeV}$, which is predicted by the quark models and by lattice QCD [25–27].

In this article, the threshold parameters and Borel parameters are taken as $s^0_{\Omega_c^*}=11.0\,\text{GeV}^2$ and $M^2=(2.5\text{–}3.5)\,\text{GeV}^2$ for the charmed baryon Ω_c^*, and $s^0_{\Omega_b^*}=45.0\,\text{GeV}^2$ and $M^2=(5.0\text{–}6.0)\,\text{GeV}^2$ for the bottomed baryon Ω_b^*. The contributions from different terms for the central values of the input parameters are presented in Tables 1 and 2, respectively. From the two tables, we can expect convergence of the operator product expansion. In the two sum rules in (11) and (12), the contributions from the terms proportional to the quark condensate $\langle\bar{s}s\rangle$ and the mixed condensate $\langle\bar{s}g_s\sigma Gs\rangle$ are suppressed due to the small mass m_s compared to the terms proportional to $\langle\bar{s}s\rangle^2$. Furthermore, from the 'full' propagator of the s quark, we can see that the mixed condensate $\langle\bar{s}g_s\sigma Gs\rangle$ is accompanied by additional large denominators, and its contribution is even smaller. On the right-

Table 1. The contributions from different terms in the sum rules for the Ω_c^* with the central values of the input parameters

	(11)	(12)
perturbative term	+80%	+83%
$\langle\bar{s}s\rangle$	+12%	+10%
$\langle\bar{s}g_s\sigma Gs\rangle$	−4%	−2%
$\langle\bar{s}s\rangle^2$	+12%	+7%
$\left\langle\frac{\alpha_s GG}{\pi}\right\rangle$	+1%	+2%

Table 2. The contributions from different terms in the sum rules for the Ω_b^* with the central values of the input parameters

	(11)	(12)
perturbative term	+78%	+80%
$\langle\bar{s}s\rangle$	+10%	+10%
$\langle\bar{s}g_s\sigma Gs\rangle$	−4%	−3%
$\langle\bar{s}s\rangle^2$	+15%	+12%
$\langle\frac{\alpha_s GG}{\pi}\rangle$	+1%	+1%

hand side of (11) and (12), the terms proportional to $\langle\bar{s}s\rangle^2$ are suppressed by the exponents $\exp\left[-m_a^2/M^2\right]$, which is balanced by the factor $\exp\left[-M_{\Omega_a^*}^2/M^2\right]$ on the left-hand side. Although the masses of the c quark and Ω_c^* baryon are much smaller than the corresponding ones of the b quark and Ω_b^* baryon, the Borel parameters M^2 are different, for the central values of the Borel parameters M^2, $\exp\left[M_{\Omega_b^*}^2/M^2-m_b^2/M^2\right] > \exp\left[M_{\Omega_c^*}^2/M^2-m_c^2/M^2\right]$. It is not unexpected that the contributions from $\langle\bar{s}s\rangle^2$ are larger in the sum rules for the Ω_b^* baryon than the ones for the Ω_c^* baryon.

If we approximate the phenomenological spectral density with the perturbative term, the contribution from the pole term is as large as (28–54)% for the charmed baryon Ω_c^* and (33–50)% for the bottomed baryon Ω_b^*. We can choose a smaller Borel parameter M^2 or a larger threshold parameters s_a^0 to enhance the contributions from the ground states. However, if we take a larger threshold parameter s_a^0, the contribution from the first radial excited state may be included; on the other hand, for a smaller Borel parameter M^2, the sum rules are not stable enough, and the uncertainty with variation of the Borel parameter is large. In the case of the multiquark states, the standard criterion of the lowest pole dominance cannot be satisfied; we have to resort to a new criterion to overcome the problem [28–31].

Taking into account all uncertainties of the input parameters, finally we obtain the values of the masses and residues of the heavy baryons Ω_c^* and Ω_b^*, which are shown in Figs. 1–4 respectively,

$$\begin{aligned} M_{\Omega_c^*} &= (2.72\pm 0.12)\ \mathrm{GeV}\,, \\ M_{\Omega_b^*} &= (6.04\pm 0.13)\ \mathrm{GeV}\,, \\ \lambda_{\Omega_c^*} &= (0.047\pm 0.008)\ \mathrm{GeV}\,, \\ \lambda_{\Omega_b^*} &= (0.057\pm 0.011)\ \mathrm{GeV}\,, \end{aligned} \tag{15}$$

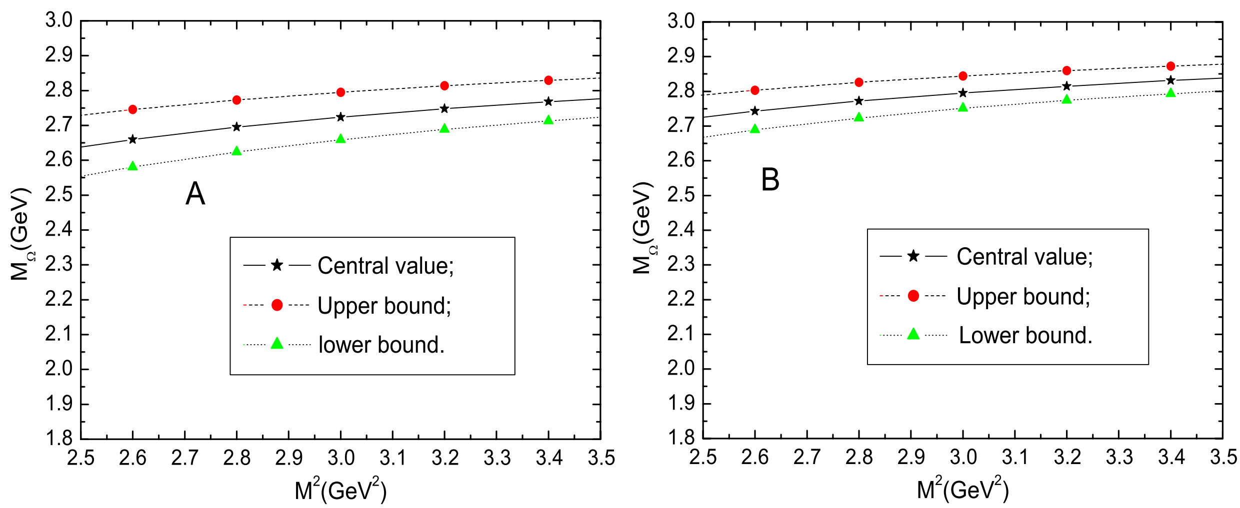

Fig. 1. $M_{\Omega_c^*}$ with Borel parameter M^2, A from (13) and B from (14)

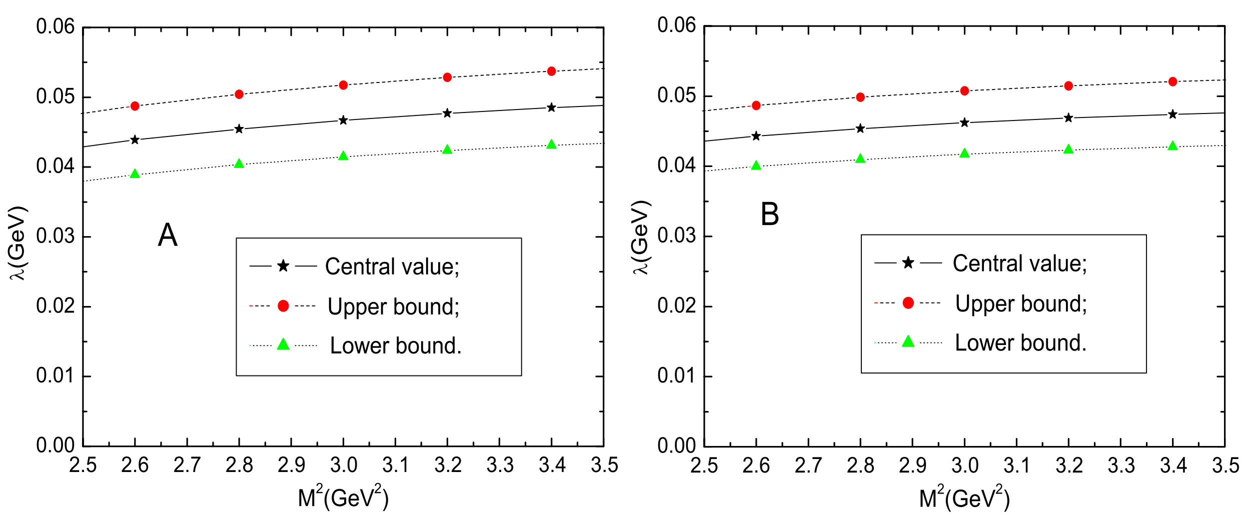

Fig. 2. $\lambda_{\Omega_c^*}$ with Borel parameter M^2, A from (11) and (13), and B from (12) and (14)

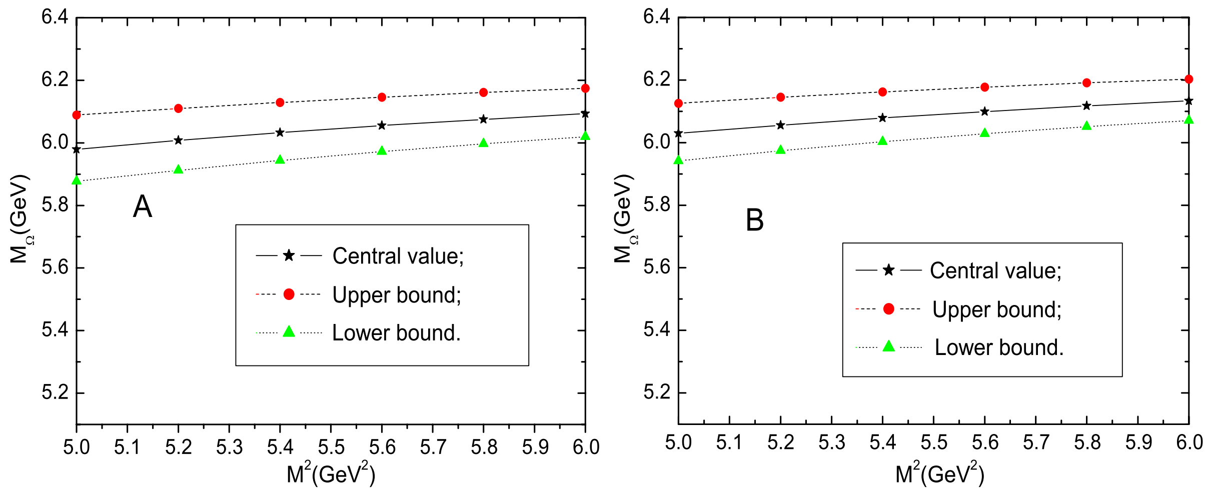

Fig. 3. $M_{\Omega_b^*}$ with Borel parameter M^2, A from (13) and B from (14)

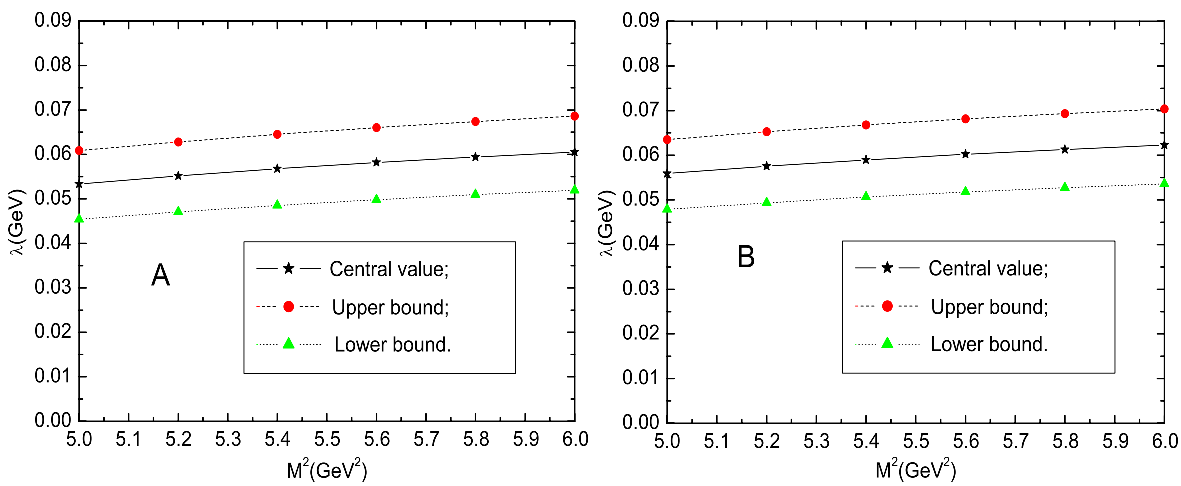

Fig. 4. $\lambda_{\Omega_b^*}$ with Borel parameter M^2, A from (11) and (13), and B from (12) and (14)

from (11) and (13), and

$$\begin{aligned} M_{\Omega_c^*} &= (2.80 \pm 0.08)\,\text{GeV}\,, \\ M_{\Omega_b^*} &= (6.08 \pm 0.12)\,\text{GeV}\,, \\ \lambda_{\Omega_c^*} &= (0.046 \pm 0.007)\,\text{GeV}\,, \\ \lambda_{\Omega_b^*} &= (0.060 \pm 0.011)\,\text{GeV}\,, \end{aligned} \tag{16}$$

from (12) and (14). The average values are about

$$\begin{aligned} M_{\Omega_c^*} &= (2.76 \pm 0.10)\,\text{GeV}\,, \\ M_{\Omega_b^*} &= (6.06 \pm 0.13)\,\text{GeV}\,, \\ \lambda_{\Omega_c^*} &= (0.047 \pm 0.008)\,\text{GeV}\,, \\ \lambda_{\Omega_b^*} &= (0.058 \pm 0.011)\,\text{GeV}\,. \end{aligned} \tag{17}$$

The value of the mass $M_{\Omega_c^*}$ is compatible with the experimental data $M_{\Omega_c^*} = (2.768 \pm 0.003)$ GeV [21], the interpolating current $J_\mu^c(x)$ can couple with the charmed baryon Ω_c^* and give a reasonable mass. The value of the mass $M_{\Omega_b^*}$ for the bottomed baryon Ω_b^* with $\frac{3}{2}^+$ is compatible with other theoretical calculations, $M_{\Omega_b^*} =$ (6.04–6.09) GeV, such as the quark models and lattice QCD [25–27]. Once reasonable values of the residues $\lambda_{\Omega_c^*}$ and $\lambda_{\Omega_b^*}$ are obtained, we can take them as basic input parameters and study the hadronic processes [32, 33], for example, the radiative decay $\Omega_c^* \to \Omega_c\gamma$, with the light-cone QCD sum rules or the QCD sum rules in an external field.

4 Conclusion

In this article, we calculate the masses and residues of the heavy baryons $\Omega_c^*(css)$ and $\Omega_b^*(bss)$ with the QCD sum rule approach. The numerical values are compatible with the experimental data and other theoretical estimations. Once reasonable values of the residues $\lambda_{\Omega_c^*}$ and $\lambda_{\Omega_b^*}$ are obtained, we can take them as basic parameters and study the hadronic processes, for example, the radiative decay $\Omega_c^* \to \Omega_c\gamma$, with the light-cone QCD sum rules or the QCD sum rules in an external field.

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